

CA6 ONJP71 R01

CA6ONJP 71R01

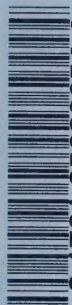
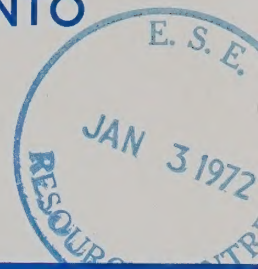
Univ. of Tor./York Univ. Joint Program in
Transportation.

Transportation Reprint No.1

OPERATIONAL ANALYSIS OF NODAL HIERARCHIES IN NETWORK
SYSTEMS. / Scott, Allen J. 1971.

UNIVERSITY OF TORONTO
YORK UNIVERSITY

JOINT PROGRAM
IN TRANSPORTATION



3 1761 05689054 4

Earth Sciences Library

OPERATIONAL ANALYSIS OF NODAL
HIERARCHIES IN NETWORK SYSTEMS

TRANSPORTATION REPRINT NO. 1

ALLEN J. SCOTT

DECEMBER 1971

THE PUBLICATION SERIES OF THE UNIVERSITY OF TORONTO—YORK UNIVERSITY
JOINT PROGRAM IN TRANSPORTATION

The Joint Program in Transportation is concerned with the promotion and coordination of interdisciplinary teaching and research in transportation at the Universities of Toronto and York. Towards this end the Program has undertaken the publication of the following four series:

1. Bi-monthly Newsletter: an informal bulletin to provide information on transportation activities within and of interest to the Joint Program.
2. Research Reports: technical reports deriving directly from research projects sponsored by the Joint Program.
3. Transportation Reprints: papers arising from research of the Joint Program and previously published in professional journals.
4. Transportation Papers: major documents such as monographs and proceedings of colloquia sponsored by the Joint Program, as selected by an editor and external review board.

Enquiries should be addressed to the University of Toronto—York University Joint Program in Transportation at either of the following addresses:

Centre for Urban & Community Studies
150 St. George Street,
University of Toronto,
Toronto 181, Ontario.

York University Transport Centre
York University
4700 Keele Street,
Downsview 463, Ontario

This publication or any part thereof may not be reproduced without the written permission of the Joint Program in Transportation at the University of Toronto and York University.

UNIVERSITY OF TORONTO - YORK UNIVERSITY
JOINT PROGRAM IN TRANSPORTATION

OPERATIONAL ANALYSIS OF NODAL HIERARCHIES IN NETWORK SYSTEMS

Allen J. Scott
Department of Geography
University of Toronto

Transportation Reprint No. 1

Reprinted from the Operational Research Quarterly, Vol. 22, No. 1, 1971 (pp. 25-37), with the permission of the author and the Operational Research Quarterly.

No further reproduction is permitted without the consent of the author and the Operational Research Quarterly.

Financial support for the University of Toronto-York University Joint Program in Transportation publication series has been provided by the Canadian Transport Commission and is gratefully acknowledged.



Digitized by the Internet Archive
in 2025 with funding from
University of Toronto

<https://archive.org/details/31761056890544>

Operational Analysis of Nodal Hierarchies in Network Systems[†]

ALLEN J. SCOTT

University of Toronto

The general location-allocation problem is identified and discussed. The hierarchical extension of this problem is then shown to be of especial interest. A computational algorithm for heuristic solution of the hierarchical problem is described. This algorithm appears to yield solutions of high quality. Finally a numerical problem is examined.

INTRODUCTION

GEOGRAPHICAL systems are very frequently susceptible to decomposition into two basic elements: A transportation or network element on the one hand, and a location or nodal element on the other hand. Certainly, this dichotomy is often detectable in those systems which are explicitly economic in character. Such systems are usually designated *location-allocation* systems (Scott¹). There is, in addition, a third dimension to these systems, and this dimension is invariably overlooked in conventional analyses of location-allocation problems. This dimension is represented by a hierarchical principle. Such a principle may apply to the transportation element of these problems, or to the location element, or, on occasions, to both elements taken simultaneously. This principle allows significant expansion and elaboration of location-allocation theory, and it corresponds at the same time to an important phenomenon frequently observable in empirical cases.

It is the purpose of the present paper to elucidate this hierarchical principle and to demonstrate its specific operation in a computable model. This model identifies a hierarchical process for a generalized node-network system.

A SIMPLE LOCATION-ALLOCATION PROBLEM FOR A NETWORK SYSTEM

Before proceeding to a direct examination of hierarchical location-allocation systems a simpler non-hierarchical model will be described as a means of fixing ideas. The hierarchical model which is subsequently described is directly derivative from this basic model.

Suppose then that there is given a network with n nodes, and that it is required to locate a number of central economic facilities at some sub-set of these n nodes. It may be assumed that each node consumes some commodity which is either produced at or transmitted through the set of central economic facilities. This system, then, is characterized by two separate kinds of costs. First, a fixed

[†] This research was supported by the Canadian Transport Commission.

capital cost which is incurred whenever a facility is actually located at any node, and, second, a transportation cost which is incurred whenever the commodity is transported through the network from some facility to some node.

Let it now be understood that it is required to organize and plan this system so that it incurs the least possible total cost. That is, it is required to locate a set of facilities and to designate a set of commodity flows from facilities to nodes in such a way that the joint costs of construction and operation of the system are the least possible. Thus, let d_{ij} represent the least cost of transportation (or shortest distance) through the network between node i and j . This cost is defined in the usual way as a cost per unit of commodity per unit of time. For the same period of time let the amortized capital cost of any facility be the constant value α . The principle here is that both of the quantities d_{ij} and α should be strictly comparable with respect to time and unit of measure, for the minimand of the problem which is considered below extends over both of these quantities jointly. Let B represent the maximum capacity of any facility, and let D_j represent the total commodity demand at the node j . Let λ_i be a binary solution variable such that whenever $\lambda_i = 1$ then a facility is located at node i , and otherwise whenever $\lambda_i = 0$. Finally, let x_{ij} be any real-valued solution variable which denotes the amount of the commodity which is transported to node j from some facility located at node i .

A simple location-allocation problem for any network system may now be written in programmatic form as follows:

Minimize:

$$Z = \sum_{i=1}^n \left(\alpha \lambda_i + \sum_{j=1}^n d_{ij} x_{ij} \right) \quad (1)$$

subject to:

$$\sum_{j=1}^n x_{ij} \leq B, \quad (2)$$

$$\sum_{i=1}^n x_{ij} = D_j, \quad (3)$$

$$M \lambda_i - \sum_{j=1}^n x_{ij} \geq 0, \quad (4)$$

$$x_{ij} \geq 0, \quad (5)$$

$$\lambda_i = \begin{cases} 1 \\ 0 \end{cases}. \quad (6)$$

Note that the number M in constraint (4) is any arbitrarily large number. Thus the constraint (4) ensures that if any positive demand, however small, is made upon the i th node then the variable λ_i is set equal to unity and this causes the emplacement of a facility at that node. The other constraints of this system are

largely self-explanatory and seem to need no further commentary here. For simplicity it has been assumed in this programme that the facilities themselves have zero unit operating costs; though this is an assumption which can obviously be relaxed without the slightest difficulty. In its present form the programme identifies both the optimal number and locations of the facilities and an optimal set of flows. This problem may be solved as a fixed-charge transportation problem (Balinski,² Kuhn and Baumol³).

One additional point concerning the problem embodied in the conditions (1)–(6) would appear to require explanation. It will be observed that no provision has been made to allow facilities to be located along the arcs of the network. Rather, all locations are by implication rigidly restricted to the nodes of the system. This restriction may be justified by the following considerations. Any facility, say facility l , exports the quantities $x_{l1}, x_{l2}, \dots, x_{ln}$. It does not matter for the moment whether these flows are or are not optimal in terms of the problem as a whole. On the other hand, for the particular *location* of the facility generating these flows to be optimal then this location must be such that the sum of the weighted distances $d_{l1}x_{l1}, d_{l2}x_{l2}, \dots, d_{ln}x_{ln}$ is minimized. The location which satisfies this criterion is known as a *median point*, and it is a theorem that in any network system this median point must always coincide with a node (Hakimi,⁴ Levy⁵). Hence, the implicit restriction that all facility locations must coincide with predefined network nodes is entirely justified.

Applications of this simple location-allocation system are not difficult to find, and the model presented above might apply with equal ease to a great variety of problems in normative economic geography. For example, the network which underlies the model might be considered to be a road or rail transportation system, while the nodes might represent population clusters. Facilities might represent such phenomena as schools, hospitals, industries or administrative offices. The problem would then be to organize the system so that its functions in the matter of location and transportation are jointly optimized. The model presented here does just that.

At the same time, a great number of different variations of this basic model is conceivable (see Scott^{1,6}). It is, moreover, possible to elaborate this model so that it is identified in relation to a continuous spatial basis as opposed to a network basis, though at the cost of greatly increased computational difficulty whenever numerical solutions are sought. Since the emphasis of the present analysis is upon the ramifications of this basic model in so far as they relate to hierarchical processes, the spatial basis of all problems to be considered below will, for simplicity, be rigidly restricted to the more tractable network basis.

A HIERARCHICAL MODEL AND ITS BASIC VARIATIONS

The nature of hierarchical node-network systems

There exists a wide variety of location-cum-transportation problems where the phenomenon or commodity undergoing distribution (or, alternatively,

collection) is most efficiently dealt with in a series of stages, and where these stages in turn identify a basic hierarchical process. Suppose, for example, that a commodity must be distributed from a set of producing centres to a set of final consumers via a number of intermediate stages. Thus, in the first stage of distribution, the commodity may be thought of as being despatched in bulk and at low unit cost from the producing centres to a series of secondary distributing centres. At these latter centres the commodity may be broken down into smaller batches and distributed at a somewhat higher unit cost to a large number of small consumer outlets. From here, the commodity finds its way to final consumers in small quantities and at a high unit cost of transportation. This process might roughly be approximated by the distribution of a commodity from factory to warehouse to retail outlet to final consumer. It will usually be the case that the capital cost associated with the construction of any individual facility will diminish as that facility occupies a progressively lower position in the total hierarchy of facilities. Conversely, it will usually be the case that the transportation cost on the commodity exported out of that facility will increase the lower the position of the facility in the hierarchy.

Depending on what kinds of technological possibilities are available, whether at the nodes or within the transportation system, this sort of devolutionary process may be an extremely efficient means of exploiting a given geographical territory. This contention is supported by the frequency with which this phenomenon is observable in empirical cases. For example, hierarchical location-allocation processes are plainly at work in the structuring of such systems as postal collection and distribution systems, electricity distribution systems, telephonic systems, waste disposal systems and, of course, in the kinds of commodity distribution systems from factory to retail establishment as already mentioned.

The succeeding account describes a basic method for the operational analysis of all such systems.

A basic hierarchical model

At the outset, it is necessary very clearly to characterize the nature of the hierarchical system which is here to be established in model form. This characterization has three basic components. First, it is assumed that each level in the total hierarchy of facilities is functionally distinct from all other levels; that is, any element in the hierarchy at level k is separate and distinct from the lower level elements $k+1, k+2, \dots$, etc. and vice versa. Second, any facility which is situated at level k in the hierarchy may receive (despatch) goods only from facilities at level $k-1$, and may despatch (receive) goods only to facilities in level $k+1$; no circumventing of levels in the hierarchy during the process of devolution is permitted. Third, it is assumed that any commodity which is transmitted through the hierarchy of facilities does not itself undergo material change at any time between its leaving some point of production and arriving

at some point of final consumption. These three conditions may appear to be somewhat restrictive. However, the means of generalizing the analysis which follows to various alternative cases will almost always be reasonably clear.

Now, assume that some transportation network is given, as before. Suppose that it is required to assign to the nodes of this network a set of central facilities graded into a hierarchy characterized by m distinctive levels of facilities. Some commodity is then assumed to be distributed to final consumers by passing first from a highest order facility through a succession of progressively lower order facilities. Let α^k denote the amortized capital cost of any facility at level k in the hierarchy. As a rule, it would be expected that $\alpha^1 > \alpha^2 \geq \dots \geq \alpha^m$. Similarly, let d_{ij}^k denote the least cost of transportation through the given network between node i and node j and where this cost, in addition, applies to each unit of commodity which is transported out of a facility of order k . Here, it would usually be assumed that $d_{ij}^1 \leq d_{ij}^2 \leq \dots \leq d_{ij}^m$. Let B^k be the capacity of any facility of order k , and let D_j be the final demand for the commodity at node j . Let λ_i^k be a binary variable denoting whether or not a facility of order k is built at node i . Lastly, let x_{ij}^k be a solution variable denoting the amount of the commodity which is transported to node j out of a facility of order k located at node i .

A hierarchical location-allocation problem can now be written.

Minimize:

$$Z = \sum_{k=1}^m \sum_{i=1}^n \left(\alpha^k \lambda_i^k + \sum_{j=1}^n d_{ij}^k x_{ij}^k \right), \quad (7)$$

subject to:

$$\sum_{j=1}^n x_{ij}^k \leq B^k, \quad (8)$$

which ensures that maximum capacities are not exceeded.

$$\sum_{i=1}^n x_{ij}^m = D_j, \quad (9)$$

which ensures that all demands are met.

$$\sum_{i=1}^n x_{ij}^{k-1} - \sum_{i=1}^n x_{ji}^k = 0, \quad (10)$$

which causes all inputs from all $k-1$ order facilities to any k order facility located at node j to be in balance with all outputs from the latter facility.

$$M\lambda_i^k - \sum_{j=1}^n x_{ij}^k \geq 0, \quad (11)$$

which is analogous to constraint (4) above, and means that whenever any demand is made upon a k order facility at node i then such a facility must actually be

constructed at that node. Lastly, the side-conditions apply:

$$x_{ij}^k \geq 0, \quad (12)$$

$$\lambda_i^k = \begin{cases} 1 \\ 0 \end{cases}. \quad (13)$$

As in the case of the programme (1)–(6), this new programme automatically determines the number of facilities which should emerge at optimality. Observe that for any given set of λ_i^k the programme reduces to an ordinary trans-shipment problem, where the commodity is trans-shipped from level to level in the hierarchy. It is important to keep this observation in mind when devising computational schemes for the solution of this problem. The model given above may now be taken as a basic prototype for a wide and miscellaneous variety of hierarchical location-allocation problems.

COMPUTATIONAL APPROACHES TO THE SOLUTION OF THE HIERARCHICAL LOCATION-ALLOCATION PROBLEM

The hierarchical problem described above is extremely difficult to resolve numerically. This results largely from the presence of fixed costs in the problem and the resulting necessity for the combinatorial variable λ_i^k . Given this combinatorial basis for the problem, an exact and fully optimal solution to the programme contained in the expressions (7)–(13) can only be guaranteed by some computationally expensive tree-searching procedure such as branch-and-bound or backtrack programming (see Scott⁶). Such a procedure would involve development of a combinatorial tree for evaluation of the binary variable λ_i^k , and co-ordinating this process with a trans-shipment problem for evaluation of the x_{ij}^k . A computational procedure of this kind would seem to be entirely out of the question for any problem of any interesting size, for these tree-searching methods typically are exceedingly demanding whether in computational labour, or in intermediate storage requirements, or in both. In a case like this it is often a good rule to compromise on the required quality of the final solution and thereby to gain a concomitant saving in computational effort. Such a compromise is established here, and in what follows a purely heuristic algorithm is developed for the solution of the hierarchical location-allocation problem.

A heuristic algorithm is any algorithm which seeks an optimal solution to some given problem but which does not necessarily guarantee a fully optimal solution. Most heuristic devices will converge to a solution in the direction of optimality, and will often yield a solution which is quite close to or even at full optimality. However, the amount of error (if any) contained in the solution can never be known on the basis of the heuristic algorithm alone. Nevertheless, the heuristic algorithm presented here for solution of the hierarchical location-allocation problem is thought to be quite powerful. This algorithm has several

features in common with a number of proven algorithms already in existence (Scott^{7,8}), and it seems to give consistently reasonable results while requiring only relatively small quantities of computational time. The rules of the algorithm, moreover, ensure unambiguous convergence of the solution process in the direction of optimality. Other heuristic algorithms for the solution of simple locational problems in network systems are described by Maranzana⁹ and Teitz and Bart.¹⁰

A heuristic algorithm for the hierarchical location-allocation problem

The algorithm which is described here involves a steady movement from a high value of the objective function, Z , to progressively lower values, and this is accomplished through a series of iterative stages each of which seeks out a locally optimal solution. Recall that Z is composed of two elements: A set of fixed capital costs activated by the λ_i^k , and a set of inter-nodal flow costs activated by the x_{ij}^k , where the latter variables are determined in a trans-shipment subproblem once the set of λ_i^k is given a numerical determination. The algorithm falls essentially into three separate phases or stages, and the description given below makes use of these essential divisions. Further, the description which is given of the algorithm assumes that all facility capacity restrictions, B^k , are inoperative and that any facility can always handle any conceivable demand which may be placed upon it. This is simply a matter of convenience for present purposes and, if required, a test of capacity feasibility can always be appended to this programme, though probably at the cost of some loss of quality in the final solution.

Stage 1. The algorithm begins by assigning to every network node a facility of every order. Thus the algorithm begins with a solution which incurs extremely low transportation costs but extremely high capital costs.

Stage 2. Now an iterative procedure is begun such that on each major pass through stage 2 of the algorithm a single facility is deleted from the problem. This deletion may be of a facility of any order. However, the deletion must simultaneously satisfy the two criteria: (a) That the deletion yields a value of Z which is lower than the value of Z yielded by any other potential single deletion, and (b) that the value of Z is strictly less than the value of the objective function for the system as it was at the start of stage 2. These two conditions ensure respectively that the solution method proceeds by local optimization and that the objective function of the problem is consistently monotonic decreasing throughout the period of operation of the algorithm. Now a return is made back to the start of stage 2 of the algorithm and the entire process repeated over again. Note, however, that when the condition (b) cannot be met then stage 2 of the algorithm is brought to a close. The algorithm now enters stage 3. A diagrammatic flow chart of stages 1 and 2 of the algorithm is shown in Figure 1.

Stage 3. This stage of the algorithm accepts as its initial input the final solution obtained in stage 2. In stage 3 of the algorithm an attempt is made to improve

upon the solution by piecewise reassignment of those facilities which remain in the problem. No facility is deleted from the problem at this stage. Rather, each facility is first removed from its specified nodal location and then is experimentally reassigned in turn to every other node. That one reassignment (if any) which produces the maximum improvement in the value of the objective function

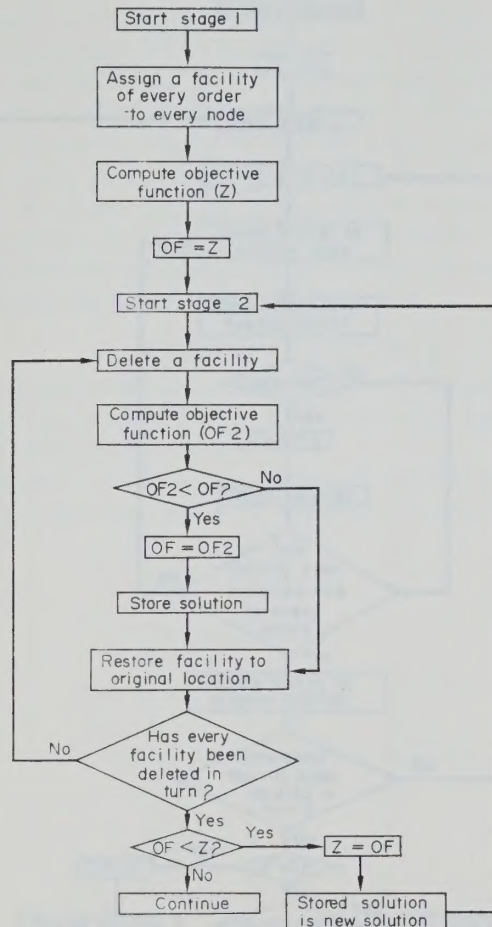


FIG. 1. Stages 1 and 2 of the heuristic algorithm.

is now accepted as the new solution. Stage 3 of the algorithm is now repeated from the beginning, and this repetition continues until no further improvements appear in the value of the objective function. Now a return is made back to the beginning of stage 2 of the algorithm, using as basic input to stage 2 the system as it is finally established in stage 3. These various iterative operations continue

until no further improvement of the system in both stage 2 and stage 3 is possible. A flow chart representing stage 3 of the algorithm is shown in Figure 2.

This heuristic algorithm combines a reasonable degree of computational efficiency with great flexibility. Stage 2 of the algorithm causes a progressive reduction in the number of facilities contained in the problem. By contrast,

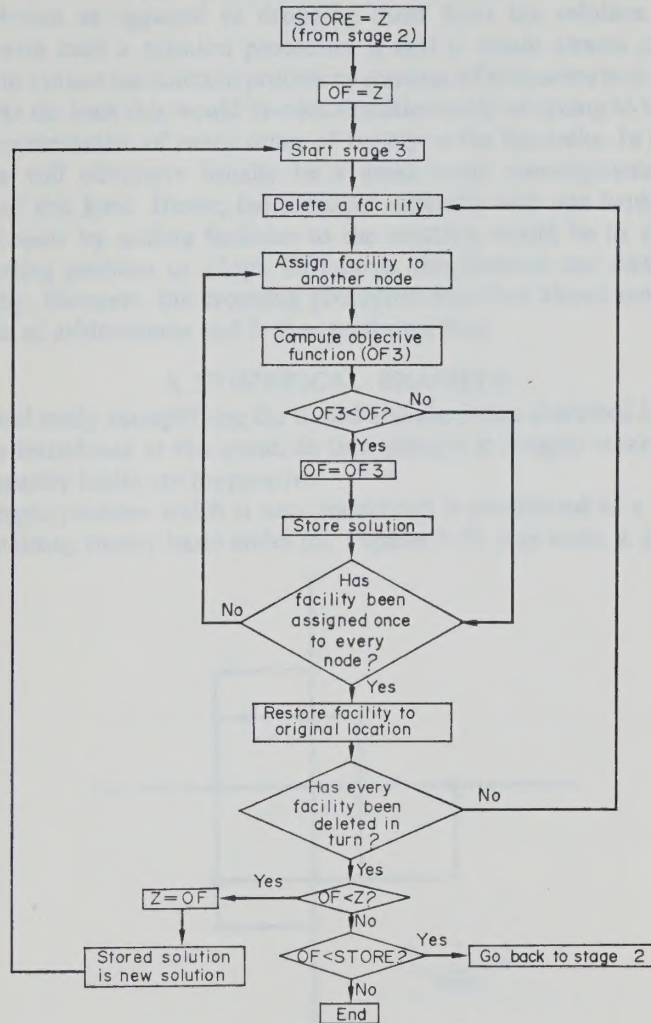


FIG. 2. Stage 3 of the heuristic algorithm.

stage 3 attempts to reshuffle the locations of a fixed number of facilities, and this reshuffling operation will often enable the algorithm to effect an exit out of any purely locally minimal solution which may have been established during

stage 2. However, it must be remarked that the final solution yielded by this algorithm can never be guaranteed to be optimal. Note, in addition, that it is entirely possible to conceive of a heuristic solution process for the hierarchical location-allocation problem which approaches optimality not from that point where all λ_i^k are equal to unity, but from a point where all λ_i^k are equal to zero. In this latter case, the algorithm would proceed by progressively adding facilities to the solution as opposed to dropping them from the solution. The only difficulty with such a solution procedure is that it would almost certainly be necessary to initiate the solution process by starting off with some non-degenerate solution. At the least this would involve simultaneously assigning to the solution set one representative of every order of facility in the hierarchy. In most problems there will obviously usually be a great many non-degenerate starting positions of this kind. Hence, the principal difficulty with any heuristic device which proceeds by adding facilities to the solution would be in deciding on which starting position to adopt. Of course, this decision can itself be made heuristically. However, the dropping procedure described above avoids at least this degree of arbitrariness and is thus preferred here.

A NUMERICAL EXAMPLE

A numerical study exemplifying the model and algorithm described in preceding sections is introduced at this point. In this example it is again assumed that all facility capacity limits are inoperative.

The sample problem which is now considered is constituted as a simple network containing twenty basic nodes (cf. Figures 3-5). Any node, j , is defined to

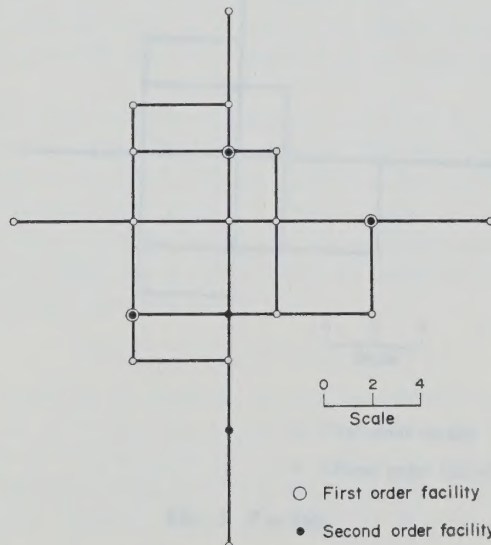


FIG. 3. $Z = 275$.

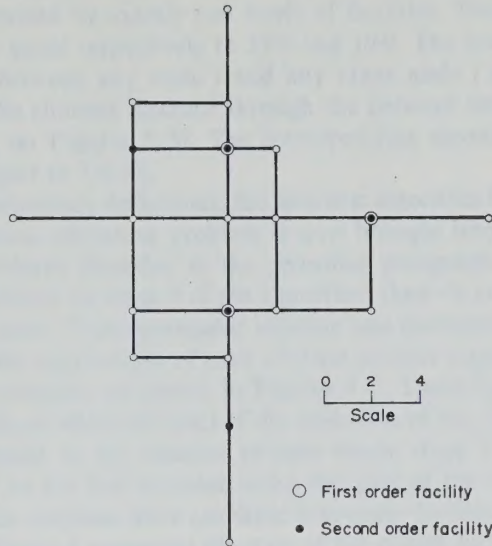


FIG. 4. $Z = 267$.

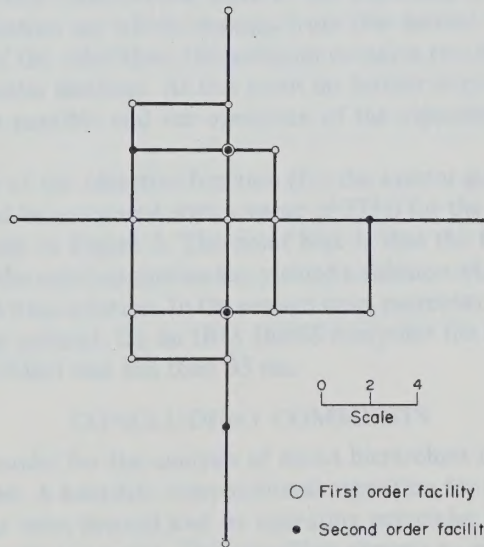


FIG. 5. $Z = 266$.

have a final demand, D_j , which is equal to unity. In addition, it is assumed that the hierarchy of facilities which is to be established to serve the nodes of the system is characterized by exactly two levels of facilities. Then, the fixed costs α^1 and α^2 are set equal respectively to 25.0 and 10.0. The first-order transportation cost, d_{ij}^1 , between any node i and any other node j is always directly proportional to the shortest distance through the network between node i and node j (see scale on Figures 3–5). The corresponding second-order transport cost, d_{ij}^2 , is set equal to $2.0 d_{ij}^1$.

Given these preliminary definitions, the heuristic algorithm for solution of the hierarchical location-allocation problem is now brought into action. To solve the particular problem identified in the preceding paragraph it was necessary to process the problem via stage 2 of the algorithm, then via stage 3 and, finally, via stage 2 once more. Three particular solution sets corresponding to the state of the system at the termination of each of these process stages seem worthy of note, and these solutions are shown in Figures 3–5. These figures demonstrate very clearly significant characteristics of the operation of the algorithm. Figure 3 represents that point in the solution process where stage 2 of the algorithm comes to an end on the first occasion when this part of the algorithm is called into action. In this solution there are three first-order facilities and five second-order facilities. Figure 4 represents the state of the system on the termination of stage 3 of the algorithm; this again is on the first major solution sequence through the algorithm, and on the completion of this phase of the solution process the algorithm returns once more to the beginning of stage 2. Figure 5 represents the solution set which emerges from this second solution sequence through stage 2 of the algorithm; this solution contains two first-order facilities and five second-order facilities. At this point no further improvement whatever of the solution is possible and the operation of the algorithm is brought to a close.

The final value of the objective function (for the system given in Figure 5) is 266.0. This should be compared with a value of 275.0 for the objective function of the system given in Figure 3. The point here is that the first major computational phase of the solution process has yielded a solution which is within about 3.4 per cent of the final solution. In the present case, moreover, this final solution is presumed to be optimal. On an IBM 360/65 computer the total solution time for this entire problem was less than 35 sec.

CONCLUDING COMMENTS

An operational model for the analysis of nodal hierarchies in network systems has been proposed. A heuristic computational algorithm for numerical solution of this model has been derived and its operating principles have been demonstrated via a numerical example. This algorithm appears to give excellent results while requiring only small to moderate quantities of computational effort to achieve a solution.

This basic hierarchical location-allocation model may now be used in the analysis of a wide variety of node-network systems. In particular the model can be used to describe and delineate the manner in which, by their nature, hierarchical locational systems become increasingly concentrated spatially as transport costs fall in relation to capital costs, and become dispersed as transport costs rise in relation to capital costs. At the same time, it seems abundantly clear that the hierarchical principle embodied in this model reflects the operation of a very widespread geographical phenomenon. For, a hierarchical ordering (whether planned or involuntarily induced) of the elements in a node-network system is often a means of exploiting a given geographical territory with maximum advantage and efficiency. The interesting question at this stage is how observed hierarchies in real node-network systems relate to the existing level of technology, and how this technology may be harnessed to extend the exploitative efficiency of the hierarchical principle. It is also of some interest to speculate as to the persistence of these hierarchical systems in observed cases, for in some instances they appear to be of diminishing importance while in others they are clearly undergoing rapid extension. These are largely problems in econometric analysis, and they are raised here as suggestions for future development of the present study.

REFERENCES

- ¹ A. J. SCOTT (1970) Location-allocation systems. *Geographical Analysis* 2, 95.
- ² M. L. BALINSKI (1961) Fixed cost transportation problems. *Naval Res. logist. Q.* 8, 41.
- ³ H. W. KUHN and W. J. BAUMOL (1962) An approximative algorithm for the fixed charges transportation problem. *Naval Res. logist. Q.* 9, 1.
- ⁴ S. L. HAKIMI (1964) Optimum locations of switching centers and the absolute centers and medians of a graph. *Ops Res.* 12, 450.
- ⁵ J. LEVY (1967) An extended theorem for location on a network. *Opl Res. Q.* 18, 433.
- ⁶ A. J. SCOTT (1969) Combinatorial programming and the planning of urban and regional systems. *Environment and Planning* 1, 125.
- ⁷ A. J. SCOTT (1969) On the optimal partitioning of spatially distributed point sets. In *Studies in Regional Science* (Ed. A. J. SCOTT), pp. 57-72.
- ⁸ A. J. SCOTT (1969) The optimal network problem: Some computational procedures. *Transpn Res.* 3, 201.
- ⁹ F. E. MARANZANA (1964) On the location of supply points to minimize transport costs. *Opl Res. Q.* 15, 261.
- ¹⁰ M. B. TEITZ and P. BART (1968) Heuristic methods for estimating the generalized vertex median of a weighted graph. *Ops Res.* 16, 901.

CA60NJP 71R01

Univ. of Tor./York Univ. Joint Program in
Transportation.

Transportation Reprint No.1

OPERATIONAL ANALYSIS OF NODAL HIERARCHIES IN NETWORK
/ Scott Allen J. 1971.

**RESOURCE CENTRE
INST. FOR ENVIRON'L. STUDIES
UNIVERSITY OF TORONTO**



